The Structure of the FITS Images produced by CRUSH and How One Should Use Them Appropriately

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1 The Primary Image: The Flux Distribution

There are four images stored in the CRUSH output image file. Each of these images reside in a separate HDU (Header-Data Unit). Most FITS aware software astronomers use (ds9, GAIA, IDL) can access the various images separately. You can also find C, JAVA or FORTRAN libraries on-line that give tools to read the data from your own code, if you prefer.

The first (Primary) image is the flux distribution map. A given pixel in that map represents a measurement value at x_i, y_i is $I(x_i, y_i)$. That is what a SHARC-2 pixel would read when looking at that given position on the sky if there was no atmosphere to dampen the signal (i.e. absorbtion corrected flux). The natural units of the flux distribution map are thus bolometer voltage units, which can be nV or V. Given, that we know what voltage signal a $1Jy$ source would produce at the bolometer (as encoded in the conversion factor that is provided to crush either via the `-mapJy` option flag or by the `MAP_V_PER_JY` configuration key), we can represent that measurement value also in approximate flux density units of $Jy/beam$, $Jy/arcsec^2$ or Jy/sr (as set by the `-unit` option or `MAP_UNIT` key).

1.1 Aperture Flux

For sources with substantial signal-to-noise ratio, the most accurate way of measuring flux is to measure the flux inside some aperture that fully encloses the source structure of interest. A flux inside such an aperture $\mathcal{A}$ can be obtained as,

$$F_{\mathcal{A}} = \frac{A_{pixel}}{(4''.85 \times 4''.77)} \times \frac{Jy}{[I]} \sum_{x_i, y_i \in \mathcal{A}} I(x_i, y_i) \tag{1}$$

where A_{pixel} is the area of a map pixel. $Jy/[I]$ is the conversion factor of map units to janskys. For maps in voltage units this is approximately given by the inverse of 798 nV/Jy.

For CRUSH default of 1/3 SHARC-2 lateral detector size map pixelization, this is simply,

$$F_{\mathcal{A}} = \frac{1}{9} \times \frac{Jy}{[I]} \sum_{x_i, y_i \in \mathcal{A}} I(x_i, y_i) \tag{2}$$

To calculate the uncertainty corresponding to this quantity, refer to Section 3.1.

1.2 Peak Flux (Flux Inside a Beam)

For point-like sources one may want to quote the flux incident inside an appropriate beam, or peak flux, as this value is likely to incorporate all flux from the point source in question. The peak flux will have units of $Jy/beam$, and is calculated as,

$$\begin{aligned} F_{peak} &\equiv \frac{A_{bolometer}}{A_{beam}} \cdot \frac{Jy}{[I]} \cdot I(x_i, y_i) \\ &= \frac{(4''.85 \times 4''.77)}{2\pi\sigma^2_{beam}} \cdot \frac{Jy}{[I]} \cdot I(x_i, y_i) \end{aligned}$$

where,

$$\begin{aligned} \sigma_{beam} &\approx 8''.5 \ FWHM \ / \ 2.35 \\ 1 \ Jy &\approx 353 \ nV \end{aligned}$$

being the typical SHARC-2 beam and flux-to-voltage conversion factor (as defined in crush.cfg).

Note, that this measure will be dependent on what beam size (and shape) you assume, and thus it is not a very well defined quantity. However, it is a flux measure that astronomers commonly quote and use, nonetheless.

To calculate the uncertainty corresponding to the flux inside a beam, refer to Section 3.2.

1.3 Smoothing Maps

The smoothing with a beam $B(x, y)$ is defined as,

$$I_{smooth}(x_i, y_i) \equiv \frac{\sum_{x'_j, y'_j} B(x'_j, y'_j) \times I(x'_j, y'_j) \ / \ \sigma^2(x'_j, y'_j)}{\sum_{x'_j, y'_j} B(x'_j, y'_j) \ / \ \sigma^2(x'_j, y'_j)} \tag{3}$$

The usual choice for the smoothing beam is a Gaussian beam of the form:

$$B(\mathbf{r}) = e^{-\frac{r^2}{\sigma^2_{beam}}} \tag{4}$$

where $FWHM = 2.35 \ \sigma_{beam}$.

Smoothing is available in the CRUSH suite via the -smooth=X option for `crush`, `show` or `imagetool`, or through the `MAP_SMOOTHING=final:X` configuration key to `crush`. All these options define a beam size that the image is convolved to, **not** convolved with. That is, if the image is already smoothed to, say, $4''$ and you want to smooth to $8''$, then a convolving beam is chosen with FWHM of $((8'')^2 - (4'')^2)^{1/2} \approx 7''$ when you use `-smooth=8.0`. This way, one has to keep in mind only what the smoothing goal is, and not what it takes to get there.

The estimated RMS for a smoothed map is discussed in Section 3.

1.3.1 A Note on Smoothing

Why smooth maps at all? There are a few important reasons in favour of smoothing with an appropriate beam. Most notably one should smooth because the map pixelization can be finer than the true physical resolution of the telescope beam. Any structure on the map that appears on a finer scale than the telescope beam is unphysical i.e., unreal. The telescope only responds to structures $\geq$ the telescope beam size, and the resulting maps should reflect that. One may also note that smoothing will produce more impressive looking maps by filtering out undesired high spatial frequency structures. Especially for faint sources that may not be easy to spot on unsmoothed maps. Yet another reason for smoothing is to do away with the malicious effect of any undetermined pointing errors that otherwise wash out a faint source's flux. E.g. smoothing with a $10''$ Gaussian in case of an $8''$ telescope beam (PSF) will properly recover point sources even if the pointing has been no better than $6''$ RMS!. (Of course, you pay the price of a higher beam noise for not having had your pointing nailed down securely.)

1.4 Filtering Undesired Extended Structures

CRUSH also offers the possibility of filtering out extended structures. The filtering is essentially the removal of the low spacial frequency components via a convolution filter. In the usual notation of convolution as an external direct product, it is:

$$I_{filtered} = I - I \otimes E(\mathbf{r}) \tag{5}$$

where $E(\mathbf{r})$ is a beam that corresponds to the typical size of extended structures above which scale the map is extensively filtered.

Note that while such a filter can be useful for identifying the location of point-like sources in the presence of undesired extended structures, the filter **will not** preserve fluxes even for point-like objects, and thus images that are thus filtered should not be used for flux extraction purposes, unless you understand what correction you will have to apply.

Filtering extended structures is also available in `imagetool` and `show` via the `-extFilter` option.

2 The Second Image: Integration Time

The second image (Integration Time) stores the total amount of time a SHARC-2 pixel has been looking at the given map position. For smoothed maps, it is reinterpreted as the total amount of time a SHARC-2 pixel has been looking over a beam centered on the given position.

Integration time has units of seconds.

3 The Third Image: RMS

The third image (RMS) represents the actual measurement uncertainty. Its value at some map pixel i located at at x_i, y_i is $\sigma^2(x_i, y_i)$, and has the same units as the Primary (Flux Distribution) image. One should think of this value as the true measurement uncertainty of SHARC-2 pixels looking over that map position, and thus it is independent of whatever smoothing that may have been applied to the map. As such it is not a very useful number unless you deal with unsmoothed maps. The map uncertainty at x_i, y_i for a smoothed map is obtained as,

$$\frac{1}{\sigma^2_{map}(x_i, y_i)} \equiv \frac{A_{pixel}}{(4''.85 \times 4''.77)} \times \sum_{x'_j, y'_j} \frac{B(x_i - x'_j, y_i - y'_j)}{\sigma^2(x'_j, y'_j)} \tag{6}$$

where $B(x, y)$ is the convolving beam function, A_{pixel} is the area of a map pixel.

The crush default is to use pixelization of 1/3 Cassegrain SHARC-2 detector lateral size, and thus, in the default case the above simplifies to:

$$\frac{1}{\sigma^2_{map}(x_i, y_i)} = \frac{1}{9} \times \sum_{x'_j, y'_j} \frac{B(x_i - x'_j, y_i - y'_j)}{\sigma^2(x'_j, y'_j)} \tag{7}$$

This can be further simplified in case of smoothing beam being Gaussian (which is most commonly used) and if the map noise is reasonably uniform inside the beam area. In this case, one arrives to the rather simple expression of:

$$\sigma_{map}(x_i, y_i) \approx 1.83 \; FWHM \times \sigma(x_i, y_i) \tag{8}$$

It is important that one understands the difference of *measurement uncertainty* vs. *map uncertainty*. The former is a characteristic of the data taking process (scan pattern) in a sense, while the latter is the value that really characterises the uncertainty of the map at some given point. One could ask the question why not put the map uncertainty as the RMS image. It would be a lot less confusing, surely... Well, not exactly. It turns out that uncertainties of all sorts (peak flux uncertainty, aperture flux uncertainty, etc.) are easily calculated from the *measurement uncertainty* but not from the *map uncertainty*. In that light the choice is obvious.

3.1 RMS for an Aperture Flux Value

For an aperture flux, calculated as discussed in Section 1.1, we can assign the uncertainty,

$$\sigma^2_{\mathcal{A}} = \frac{A_{pixel}}{(4''.85 \times 4''.77)} \sum_{x_i, y_i \in \mathcal{A}} \sigma^2(x_i, y_i) \tag{9}$$

which, for the default CRUSH map pixelization of 1/3 of SHARC-2 detector lateral size, simplifies to:

$$\sigma^2_{\mathcal{A}} = \frac{1}{9} \sum_{x_i, y_i \in \mathcal{A}} \sigma^2(x_i, y_i) \tag{10}$$

3.2 RMS for Flux Incident Inside a Beam

The uncertainty for a flux quoted inside a beam is of identical for to the previously discussed map uncertainty (Section 3, except that the smoothing beam is replaced by the beam over which the flux is measured. For an $8''.5$ FWHM Gaussian telescope beam (PSF) under the default map pixelization,, in a region of reasonably uniform noise, this simplybecomes,

$$\sigma_{beam}(x_i, y_i) \approx 15.6 \; \sigma(x_i, y_i) \tag{11}$$

3.3 Excess RMS

All calculated uncertainties (measurement uncertainty, map uncertainty etc.) are estimated under the assumption of independent pixels. What happens if there is a correlation of the noise residuals among pixels? Suppose, the bolometer signal can be written as,

$$I(t,b) = ... + n(t,b) + \sum_{b' \neq b} C(b,b') n(t,b') \tag{12}$$

where $n(t,b)$ is the bolometer noise and $C(b,b')$ is a correlation coefficient between two bolometers b and b'. In this case a generalized linear combination of the bolometer signals will be of the form

$$A = \sum_b \alpha_b I(t,b) \sim ... + \sum_b \alpha_b n(t,b) + \sum_{b' \neq b} C(b,b') n(t,b') \tag{13}$$

The corresponding uncertainty will be simply the expectation value of the noise terms squared. Which in the case of residual covariances $C(b,b') << 1$ and comparable detector noise $\sigma_b \approx \sigma_{b'}$, can be reduced as,

$$\begin{aligned}
\sigma_A^2 &= \sum_b \alpha_b \, \langle \, \left(n(t,b) + \sum_{b' \neq b} C(b,b') n(t,b') \right)^2 \, \rangle_{t,b} \\
&= \sum_b \alpha_b \, \langle \, n^2(t,b) + \sum_{b' \neq b} C(b,b') n(t,b) n(t,b') + \mathcal{O}(C^2) \, \rangle_{t,b} \\
&\approx \sum_b \alpha_b \, \sigma_b^2 \left\{ 1 + \sum_{b' \neq b} C(b,b') \right\} \\
&= (1 + \mathbf{C}) \times \sum_b \alpha_b \, \sigma_b^2 \\
&= (1 + \mathbf{C}) \times \hat{\sigma}_A^2
\end{aligned}$$

That is, the true measure of the uncertainty will be a factor of $(1+C)$ higher than in the case of independent pixels.

The map fluxes in the flux image are also just a linear combination of pixel data, and thus the same factor applies. If one wants to have a reliable estimate of the flux uncertainty, the statistically calculated errorbars have to be scaled upward by a factor of $(1+C)$ to account for pixel-to-pixel covariances. In case of crush, one should be able to determine the scaling factor as a χ^2 over the areas of the map not containing sources, defined as,

$$(1 + C) \rightarrow \chi^2 \equiv \sum_{x_i, y_i} \frac{F^2(x_i, y_i)}{\sigma_{map}^2(x_i, y_i)} \tag{14}$$

Alternatively, in the presence of sources (though may be faint) one can fit a Gaussian to the signal-to-noise distribution of the map pixels (obtainable via the `histogram` utility) in the appropriate range where the distribution is well aproximated by a Gaussian. The apparent with of the fitted Gaussian distribution will provide the noise scaling factor.

One can apply the appropriate scaling factor to CRUSH maps via the `-rmsscale` option flag of `imagetool`, so that all calculated noise or signal-to-noise estimates are realistic afterwards.

4 The Fourth Image: Signal-to-Noise

The signal-to-noise at (x_i, y_i) for a smoothed map is defined as

$$S/N(x_i, y_i) = \frac{F_{x_i,y_i}}{\sigma^2_{map}(x_i, y_i)} \tag{15}$$

While it is purely a derivative of the images already discussed, it has been appended to the reduced FITS file, to ease the job of the astronomer. It requires some manipulation, either with the help of some custom written code or via some image manipulating environment such as IDL, to calculate the map uncertainty from the measurement uncertainties, as previously discussed. Thus the signal-to-noise image was made available so that the observer can quickly and easily evaluate the significance of a peak flux measurement on the map. Recall that the actual noise is some factor of $\mathbf{2}$ or $(\mathbf{1} + \mathbf{C})$ times higher than the estimated noise to the the not fully uncorrelated nature of detector residuals. Accordingly, the true signal-to-noise is the same factor lower than the estimate thus available.